# Modelling and Simulation of DC-Motor Electric Drive Control System with Variable Moment of Inertia

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Abstract: - This work represents a mathematical analysis and simulation of dc-motor electric drive control system with variable moment of inertia. A separately-excited dc motor is used in this control system. A mathematical model for this motor has been simulated and tested in Matlab/Simulink. A closed-loop control system for this dc electric drive system is proposed. The proposed control system is based on the technical optimum method of design. The controlled variable of this system is the load angular speed. In this control system the moment of inertia is considered to be variable. It varies as a function of time. A speed controller and a current controller are designed for the suggested model to meet the desired performance specifications by using the technical optimum method. These controllers are attached to the control system and the closed-loop response is observed by simulation and testing this model. The results show the high-performance of the designed control system.

*Index Terms* - dc motor drive, variable moment of inertia; speed control; simulation.

#### I. Introduction

The separately excited direct current motors with conventional proportional integral (PI) speed controller are generally used in industry. Many works paid big attention to dc motor drives and their control [1, 2, 3, 4, 5]. In [6] author is presenting a design of speed controller for speed control of converter fed dc motor drive using model order reduction technique. The work [7] studied the speed control of separately excited dc motor. Vichupong and Bayoumi studied systems with PI and PID controller [4, 8].

In recent decades, a new framework for the design of control systems has emerged. Its development was prompted in the 1960s by two factors. First was the arrival of interactive computing facilities which opened new avenues for design, relying more on numerical methods. In this way, routine computational tasks, which are a significant part of design, are left to the computer, thus allowing the designer to focus on the formulation of the design problem, which requires creative skills. This has led to a major shift in the field of design, with more emphasis placed on general principles for the formulation of design problems. Second is a shift in the aims of control theory from the aspect of designing a system with a good margin of stability to designing a system such that all control variables are kept within specified tolerances regardless of any disturbances to the system.

Works of well known Nyquist, have been influential in forming the foundation of what is now the mainstream theory

for the design of control systems, with its remarkable successes and, as will be seen, some significant limitations. These works introduced two key ideas, respectively called stability and sensitivity, which constitute the foundation of the conventional framework for control systems design.

In classical control system, the stabilization of linear time invariant system is achieved by state variable feedback technique or selection of PID controller or phase compensator. The design of controllers and compensators for higher order system involves computationally difficult and cumbersome tasks. Hence there is a need for the design of a higher order system through suitable reduced order models [6, 9, 10]. The controller designed on the basis of reduced order models should effectively control the original higher order system [11]. In the work [11] author is analyzing a control system with ac-motor with a variable moment of inertia. But author did not study such control system with dc motor.

In general, an accurate speed control scheme requires two closed-loops, an inner current control loop [12, 13] and an outer speed control loop.

All the above mentioned works did not study a dc-drive control system with variable moment of inertia. The main purpose of this work is to present a mathematical analysis and simulation of dc motor and then to get a model for a control system with this dc-motor electric drive with variable moment of inertia. The speed of separately excited dc motor can be controlled from below and up to rated speed using chopper as a converter. The chopper firing circuit receives signal from controller and then chopper gives variable voltage to the armature of the motor for achieving desired speed. There are two control loops, one for controlling current and another for speed. The controller used is proportional-integral type which provides fast control. Modelling of separately excited dc motor is done. The complete layout of dc drive mechanism is obtained. The designing of current and speed controller is carried out. The optimization of speed controller is done using modulus hugging approach, in order to get stable and fast control of dc motor. After obtaining the complete model of dc drive system, the model is simulated using MatLab. The simulation of dc motor drive is done and analyzed under varying speed and varying load torque

When the machine is made to run from zero speed to a high speed then motor has to go to specified speed. But due to electromechanical time constant motor will take some time to speed up. But the speed controller used for controlling

speed acts very fast. Speed feedback is zero initially. So this will result in full controller output and hence converter will give maximum voltage. So a very large current flow at starting time because back emf is zero at that time which sometime exceeds the motor maximum current limit and can damage the motor windings. Hence there is a need to control current in motor armature. To solve the above problem we can employ a current controller which will take care of motor rated current limit. The applied voltage will now not dependent on the speed error only but also on the current error. We should ensure that the armature voltage is applied in such a way that machine during positive and negative torque, does not draw more than the rated current. So, an inner current loop hence current controller is required.

#### II. THE MATHEMATICAL MODEL OF THE DC MOTOR

The dc-motor can be mathematically modelled by using the dynamic equivalent circuit of dc motors. The voltage equation of the armature circuit under transient is given by the following equation:

$$\upsilon = R_a i_a + L_a \frac{di_a}{dt} + K_e \phi \, \omega_m \tag{1}$$

Where:  $\upsilon$  - the source voltage,  $i_a$  - the motor armature

current,  $K_e\phi\,\omega_m$  - the back emf.

From the dynamics of motor load system:

$$J\frac{d\omega_m}{dt} = T - T_L - B\omega_m \tag{2}$$

B - the coefficient of viscous friction,

J - the moment of inertia of the motor load system referred to the motor shaft,

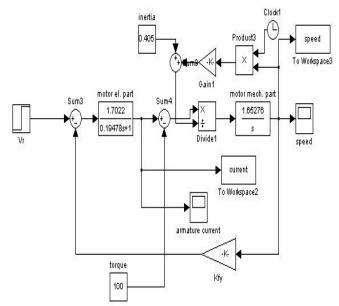
T ,  $T_L$  - are the motor electromagnetic torque and the mechanical torque of the load, respectively.

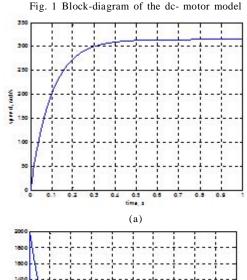
### III. SIMULATION OF THE DC MOTOR

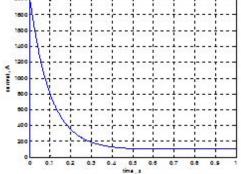
Basing on the equations (1) and (2) the block-diagram shown in Fig. 1was created for simulation the dc motor. Some results of simulation are depicted in Fig. 2. These curves show the online direct starting of the motor at different conditions of operation (full load, no-load). This model will be used in the control system.

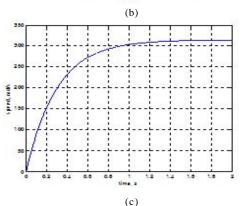
### IV. THE CONTROL SYSTEM MODEL

Every control system is designed for a specific application and therefore should be known as the performance criteria. The desired specifications are usually translated in the form of a rational transfer function called reference model. Given a process whose performance is unsatisfactory and the reference model having the desired performance, a controller is designed such that the performance of overall system









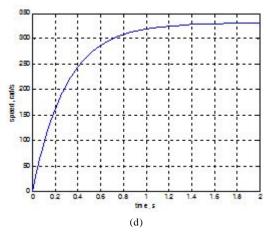


Fig. 2 (a) Speed versus time of the dc motor at full-load (b) current versus time of the dc motor at full-load (c) Speed versus time of the dc motor at full-load at higher moment of inertia (d) Speed versus time of the dc motor at no-load at higher moment of inertia

matches the reference model.

Yeung presents graphical design method for common continuous-time and discrete-time compensators [14]. The method is based upon a set of Bode design charts which have been generated using appropriately normalized compensator transfer functions. In [15-25] authors presented reduction of high order models of control systems, particularly pole-zero cancellation method.

In general, series controllers are preferred over feedback controllers because for higher order systems, a large number of state variables would require large number transducers to sense during feedback. This makes the use of series controllers very common.

In the current paper series controllers are used, and reduction of high order control system is presented, using technical optimum method, which is similar to pole-zero cancellation method. Figure 3 shows the block-diagram of this control system. This control system has two loops; the current loop and the speed loop. Controllers have been designed for this control system to satisfy the required performance using technical optimum method [23].

In the studied control system the used motor is a dc motor. The load of this motor is considered with a variable moment of inertia. Moment of inertia is varying as a function of time. The dc motor model discussed above has been used in the current electric-drive control system. This model has a block-diagram, which contains two parts: electrical part and mechanical part. The electrical component has the following transfer function:

$$G_a(s) = \frac{1}{R_{a.t} \left( s \cdot \frac{L_{a.t}}{R_{a.t}} + 1 \right)} = \frac{K_a}{T_a \cdot s + 1}$$

where

$$K_a = \frac{1}{R_{a.t}} \quad ; T_a = \frac{L_{a.t}}{R_{a.t}}$$

 $\boldsymbol{L}_{at},\,\boldsymbol{R}_{at}$  - the total inductance and resistance of the armature circuit,

T<sub>a</sub> – the electrical time constant of the armature circuit. The mechanical component of the motor has the gain:

$$G_m(s) = \frac{K\phi}{J \ s} = \frac{K_m}{s}$$

The used converter consists of two series connected elements: the power part and the control-circuit element. Each of them may be considered as a first order system:

$$G(s) = \frac{K}{T \cdot s + 1}$$

And the two systems may be approximated as one first order system, time constant which is equal to the sum of their time constants. The control part has an inertia factor  $T_{\phi}$ , that caused by RC filter. The power part of the converter has also inertia factor  $(T_{\alpha})$ , which depends on the grid frequency. So, the gain of the converter has the form:

$$G_c(s) = \frac{K_c}{T_c \cdot s + 1}$$

Where T<sub>c</sub> - the converter time constant.

The block diagram of the rectifier and the electrical part of the motor may be considered as two series elements.

To get the technical optimum in the system, a PI controller may be used as current regulator with the form:

$$G_{c.r,t.o}(s) = \frac{1/K_{fb,c}}{G_{c.l}(s) \ 2 \ T_{c}(T_{c}+1)} = K_{c.r} + \frac{1}{T_{c.r}.s}$$

The object of control for the speed loop is the current loop, which is at technical optimum has the gain:

$$G_{c.l,t.o}(s) = \frac{1/K_{fb,c}}{1 + 2T_c \cdot s + 2T_c^2 \cdot s^2} \approx \frac{1/K_{fb,c}}{2 \cdot T_c \cdot s + 1}$$

Where  $K_{fb,c}$  – the coefficient of the current feedback.

And so the transfer function of the current loop and the mechanical part of the motor, connected in series, has the form (which is the object of the control for the speed loop):

$$G_{o.r,s}(s) = \frac{1/K_{fb,c}}{2T_c \cdot s + 1} \cdot \frac{K_m}{s}$$

For technical optimum adjustment the transfer function of the speed regulator must be a P-controller with the form [26]:

$$G_{s.r,t.o}(s) = \frac{1/K_{fb,s}}{G_{o.r.s}(s) \ 4 \cdot T_c \cdot s \ (2 \cdot T_c \cdot s + 1)} = K_{s.r.}$$

V. SIMULATION AND ANALYSIS OF THE DESIGNED CONTROL SYSTEM

Fig. 3 illustrates the block-diagram created basing on the above mentioned design of the dc electric drive control system. The used controllers are; PI-controller in the current loop and P-controller in the speed loop. In this model the moment of inertia is varying as a function of time. A numerical example for this system was carried out for simulation using Matlab/Simulink. This case study is considered with variable

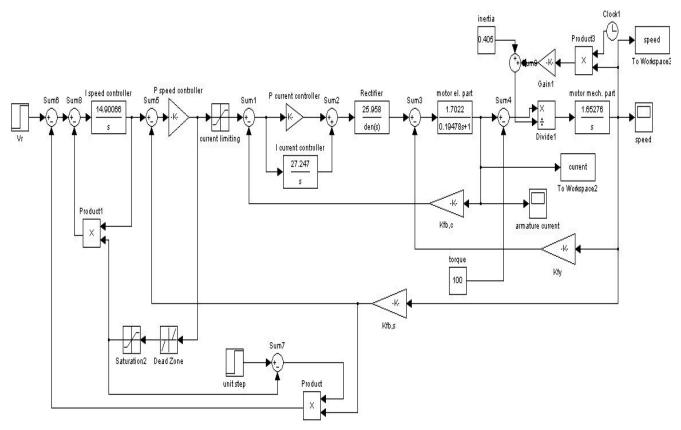


Fig. 3 Block-diagram of the dc electric drive control system

moment of inertia at different conditions. Such cases may be used in some industrial applications like winding-up drives of different kinds of tapes.

Some results of simulation are illustrated in Fig. 4, 5 and 6. These curves show the transients and steady-state operation of this control system at different conditions. Curves in Fig. 4a and 4b illustrate the angular speed and the current versus time, respectively, with variable moment of inertia. An examination of this Fig. shows that the settling time is about one second, the overshoot is 1.5% and error in speed value is less than 0.04%. The starting current is limited to the desired value 200% of the rated. Curves in figure 4c and 4d illustrate other value of the angular speed and the current versus time, respectively, with variable moment of inertia. The curves show that the settling time is about 0.9 second, the overshoot is about 3% and error in speed value is less than 0.03%.

Curves in Fig. 5a and 5b present the angular speed and the current versus time, respectively, with variable moment of inertia at full-load conditions. Curves depicted in Fig. 6a and 6b illustrate the angular speed and the current versus time, respectively, with constant moment of inertia at full-load conditions.

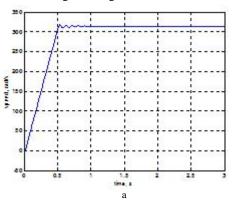
The proposed model has been tested with wide range of change in the load and moment of inertia and it showed high performance in both transients and steady-state.

## Conclusion

A mathematical model for the dynamic simulation of the dc motor has been developed. This model has been verified

by simulation using Matlab. The developed model describes the operation of the system under transients and steadystate conditions. The system was tested by simulation with different values of load torque on the shaft of motor, with constant moment of inertia and with variable moment of inertia.

The proposed control system is based on the technical optimum method of design. The controlled variable of this system is the load angular speed. In the developed electric drive control system the moment of inertia is variable. It is considered to vary as a function of time. A controller is designed for the reduced order model to meet the high performance specifications. This controller is attached to the closed loop control system. Results of simulation of the suggested model at different conditions of operation show that the proposed control system satisfied the required high performance of control system, even when the moment of inertia is increasing with high rate.



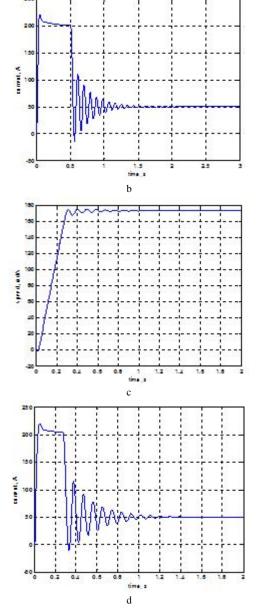
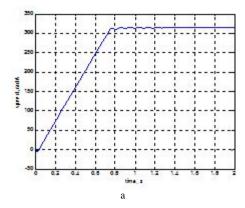


Fig. 4 (a) Speed versus time of the dc drive control system at half-full load (b) current versus time of the dc drive control system at half-full load (c) Speed versus time of the dc drive control system at half-full load (d) current versus time of the dc drive control system at half-full load



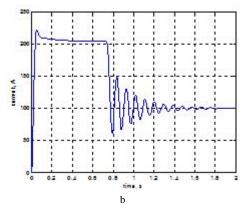


Fig. 5 (a) Speed versus time of the dc drive control system at full load with variable moment of inertia (b) current versus time of the dc drive control system at full load with variable moment of inertia

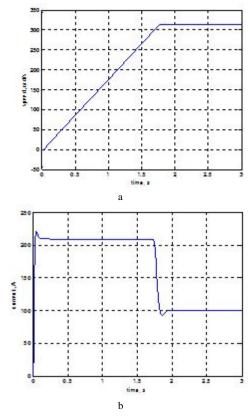


Fig. 6 (a) Speed versus time of the dc drive control system at full load with constant moment of inertia (b) current versus time of the dc drive control system at full load with constant moment of inertia

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